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1D NUMERICAL MODEL OF DELTA RESPONSE TO RISING SEA LEVEL

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ABSTRACT Rivers form deltas wherever they flow into standing water such as a lake, a reservoir or the ocean. Under conditions of constant base level of standing water deltas can be expected to gradually prograde outward, so that the delta shoreline regresses "seaward." Rising base level such as that which prevailed in the ocean at the end of the last glaciation, however, can not only slow this progradation, but reverse it, so that the shoreline migrates landward, or transgresses. An extreme limit of this case is one of shoreline starvation, for which the supply of sediment at the shoreline drops to zero and the delta goes into transgressive autoretreat. A numerical model is developed to study delta evolution, including autoretreat, and is compared against a set of experiments. The model also encompasses the migration of a bedrock-alluvial transition at the upstream end of the delta.

INTRODUCTION

How do river deltas respond to changing base level, i.e. variation in the elevation of the ponded water into which they flow? Water surface elevation in lakes and reservoirs is often subject to variation. The ocean itself does not maintain a constant elevation. The melting of continental ice sheets over a 12,000-year period commencing with the end of the Pleistocene ice age and ending about 8,000 years ago caused a sea level rise of about 120 m. Until recently sea level has been relatively constant since then, but global warming appears to be causing further sea level rise.

As river-borne sediment reaches standing water, a fluvial topset deposits out and an avalanching foreset forms subaqueously. Under conditions of constant base level (elevation of standing water), the delta can be expected to prograde into the body of water. As base level rises, however, it creates extra accommodation space that must be filled in order for the delta to prograde, i.e. for the delta shoreline to move outward (regress). Base level rise can cause the shoreline to prograde less rapidly than it would with a constant base elevation. Indeed, base level rise can cause the shoreline to retreat landward (transgress). Muto and Steel (1992) have proposed a type of transgression that they have named "autoretreat," during which so much sediment is consumed in filling the topset that the sediment transport rate drops to zero at the shoreline. Under this condition the foreset ceases to build, and becomes a relict as the shoreline retreats upstream from it.

The concept of autoretreat can be explained in the context of Figure 1. Figure 1a shows the case of constant base level ξ. Water and sediment are released at the upstream end of the reach over a bedrock basement with slope S_{base}. This slope is taken to be sufficiently high for the sediment to transported below capacity over the bedrock basement. The effect of standing water downstream, however, causes sediment to deposit out and form an alluvial topset at slope $S < S_{base}$ corresponding to capacity sediment transport. The alluvial bed slope decreases downstream as sediment deposits out. The sediment load remaining at the shoreline (topset-foreset break) is supplied to an avalanching subaqueous foreset, which progrades outward over the basement. As can be seen in Figure 1a, there are three moving boundaries: the bedrock-alluvial break migrates upstream; the topset-foreset break (shoreline) progrades downstream (regresses); and the foreset-basement break also progrades downstream into ever-deeper water.

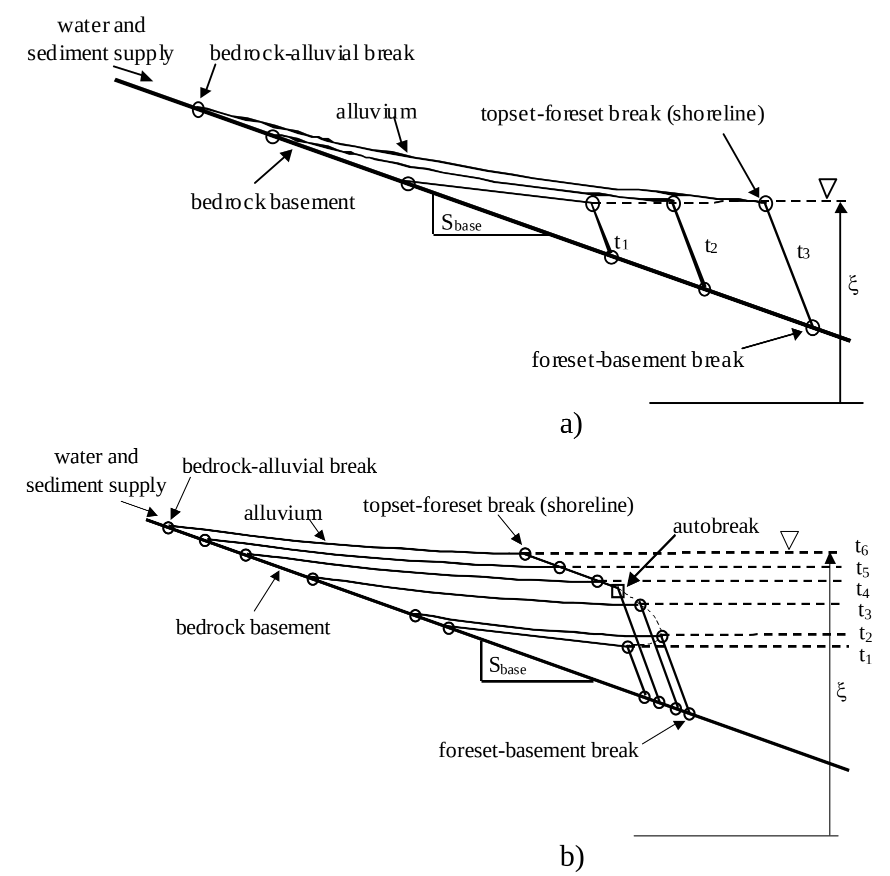

Figure 1. a) Delta evolution with constant base level; b) delta evolution with rising base level

In Figure 1b the configuration is the same as that of Figure 1a except that base level is rising in time. In the early stages of delta development, the three moving boundaries all move in the same directions as in the case of Figure 1a. The river must fill the topset space created by rising sea level, however, so this causes the sediment transport to decrease more rapidly downstream, leaving less and less sediment delivery at the shoreline to supply to the foreset. As a result, the shoreline progrades (regresses) ever more slowly, until it reverses and moves upstream (transgresses). At some point, i.e. "autobreak," there is no longer any sediment left at the shoreline to supply to the foreset. Beyond the time of autobreak the delta goes into autoretreat, with the entire delta shifting upstream away from a relict foreset.

Muto and Steel (1992, 1997) have proposed that sea level rise at the end of the last glaciation forced many deltas into autoretreat. Muto (2001) performed a series of simple laboratory experiments in order to demonstrate the concepts of autobreak and autoretreat. Here these experiments are augmented and explained with a mechanistic formulation and a predictive numerical model.

EXPERIMENTS OF MUTO

Muto (2001) considered the simple 1D configuration sketched in Figure 1b. Water and sediment were introduced into a narrow channel with an inerodible basement. The basement had constant slope S_{base} and slope angle $\theta_{base} = \tan^{-1}(S_{base})$. The basement slope was chosen to be sufficiently steep so that the sediment was transported below capacity over it, with negligible sediment deposition. The basement thus served as a simple model for bedrock. Ponded water was maintained at the downstream end, and was allowed to rise at the constant rate $\dot{\xi}$. This ponding forced the flow to decelerate and the sediment to deposit out on the bed, forming an alluvial topset and an avalanching foreset.

Before enumerating the experimental conditions in detail, it is useful to study Figure 2, which shows the delta configuration at the end of one of the experiments. It can be seen in the image that the shoreline first prograded (regressed), and then transgressed until autobreak was reached. Beyond autobreak the topset decoupled from and migrated upstream of the foreset, which was left as a relict.

The experiments of Muto (2001) were conducted in a narrow tank with a length of 2 m and a depth of 1 m. The width of the tank was either 5 or 10 mm for the experiments. This configuration allowed for a simple 1D delta without complications in the transverse direction. (Muto and Steel, 2001, have, however, demonstrated the concept of autoretreat in the case of 2D deltas with transverse flare.) The sediment used in the experiments was a uniform fine quartz sand with a median and geometric mean size of 0.212 mm, a geometric standard deviation of 1.24 and a specific gravity near 2.65.

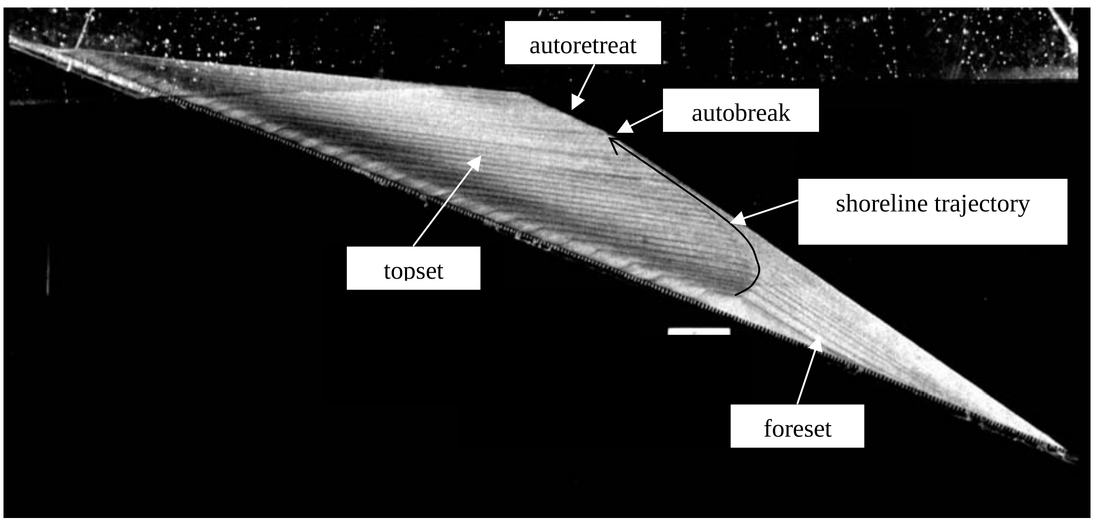

Figure 2. Image from the end of one of the experiments of Muto (2001). Flow was from left to right.

The 29 experiments in Table 1 of Muto (2001) are considered here. Basement slope angle θ_{base} varied from 11.9° to 31.2°; the angle of repose θ_r of the sediment was found to be near 35°. Water discharge per unit width q_w took one of the two values 2.18 cm^2/s and 4.36 cm^2/s. Volume sediment feed per unit width q_{sf} (sediment + pores) ranged from 0.297 cm^2/s to 1.093 cm^2/s. The rate of base level rise $\dot{\xi}$, which was held constant for the duration of each experiment, varied from 0.126 mm/s to 0.387 mm/s. As can be seen in Figure 2, a small amount of black carborundum powder was occasionally added to allow visualization of the structure of the deposit.

Any attempt at a mechanistic numerical model of the experiments of Muto (2001) requires an appropriate sediment transport relation. Although many such relations are available in the literature, the experiments of Muto (2001) do not fit the conditions of these relations, in that a) the volume concentration of sediment in the slurry fed into the flume was very high, i.e. on the order of 10 percent and b) flow depth was very low, i.e. on the order of 1 mm. As a result the data from the experiments were used to develop such a relation. Muto (2001) recorded the bed slope angle θ_u at the upstream end of the alluvial deposit, where the transition from bedrock to alluvial conditions is made. The bed slope $S_u = \tan(\theta_u)$ thus corresponds to the slope at which the sediment feed q_{sf} is just transported at capacity by the flow discharge q_w. With this in mind a simple dimensionless relation of the following form was hypothesized for sediment transport; where q_s denotes the local volume sediment transport rate per unit width (sediment + pores) and S denotes the local bed slope,

$$\frac{q_s}{q_w} = \alpha S^n \quad (1a)$$

Based on the data of Muto (2001), the following values of α and n were determined by regression, as shown in Figure 3;

$$\alpha = 12.3, \quad n = 2.24 \quad (1b,c)$$

The coefficient of correlation R^2 is given in the figure.

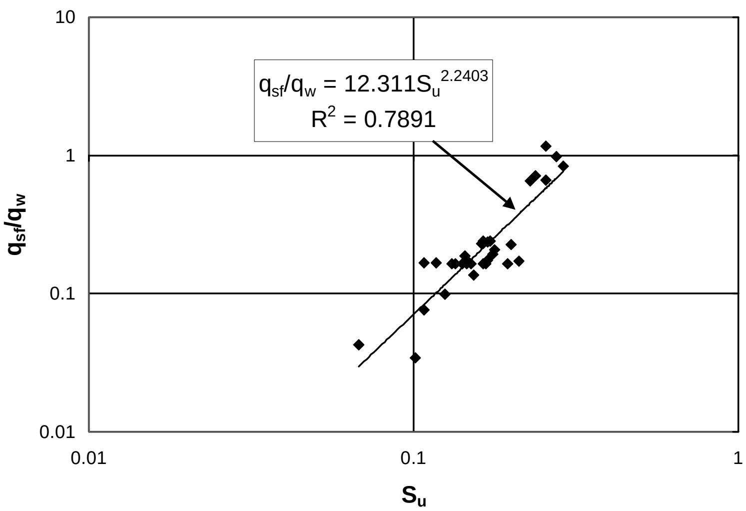

Figure 3. Empirical sediment transport relation for the data of Muto (2001).

Equation (1) is not intended to be of any general significance. All that is required here is that it adequately explain sediment transport in the facility of Muto (2001) for the purposes of the numerical model described below. In any application of the numerical model to field conditions (1) can be replaced by one of many equations that are known to be more appropriate.

FORMULATION OF THE NUMERICAL MODEL

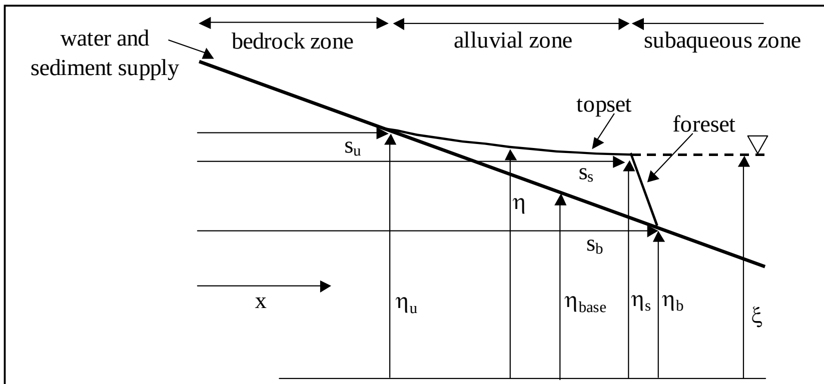

Figure 4. Definition diagram.

The numerical model described below is a descendant of the work of Swenson et al. (2000) and Kostic and Parker (2003a,b). These models allow for two moving boundaries; the topset-foreset break (shoreline) and the foreset-bottomset break (foreset-basement break in the present case). The present analysis, however, allows for a third moving boundary, i.e. the bedrock-alluvial transition point shown in Figures 1 and 4.

The key parameters of the analysis are defined as follows and illustrated in Figure 4: η = alluvial bed elevation; η_{base} = elevation of basement; x = streamwise distance; t = time; S_{base} = constant slope of the basement on which the sediment deposits; S_a = constant slope of avalanche onto the foreset ($S_a > S_b$), here set equal to the tangent of the angle of repose θ_r of 35°; s_u = streamwise position of the bedrock-alluvial break; s_s = streamwise position of the topset-foreset break (shoreline); s_b = streamwise position of the foreset-bottomset break; η_u = elevation of the bedrock-alluvial break; η_s = elevation of the topset-foreset break; η_b = elevation of the foreset-bottomset break; q_s = volume sediment transport rate per unit width (including pores); ξ = base level elevation; and $\dot{\xi}$ = time rate of base level rise, here always positive. Alluvial bed slope S is given as

$$S = -\frac{\partial \eta}{\partial x} \tag{2}$$

The Exner equation of sediment conservation can be written as

$$\frac{\partial \eta}{\partial t} = -\frac{\partial q_s}{\partial x} \tag{3}$$

(Note that q_s includes both sediment and pores; in the experiments under consideration the porosity was near 0.5.) The boundaries s_u, s_s and s_d are all moving boundaries that change in time. The delta builds out over a set basement with elevation profile $\eta_{base}(x)$ and constant slope S_{base}. The alluvial zone of the delta begins at the the bedrock-alluvial break with an elevation equal to that of the basement, and ends at the elevation of the standing water. The boundary conditions on (1) at the upstream end of the alluvial zone are a sediment feed condition;

$$q_s\big|_{x=s_u} = q_{sf} \tag{4}$$

where q_{sf} denotes the feed value of q_s, and a continuity condition matching the bedrock zone smoothly with the alluvial zone;

$$\eta_u \equiv \eta[s_u(t), t] = \eta_{base}[s_u(t)] \tag{5}$$

Equation (5) is further reduced by taking the time derivative, resulting in the relation

$$\dot{s}_u = -\frac{1}{S_{base} - S_u} \frac{\partial \eta}{\partial t}\bigg|_{s_u} \tag{6a}$$

where the dot denotes a time derivative and S_u denotes the upstream alluvial bed slope;

$$S_u = -\frac{\partial \eta}{\partial x}\bigg|_{s_u} \quad , \quad S_{base} = -\frac{\partial \eta_{base}}{\partial x} \tag{6b,c}$$

Because S_{base} can be expected to be higher than S_u, (6a) ensures that bed aggradation causes the bedrock-alluvial transition to move upstream.

A continuity condition similar to (5) holds at the shoreline;

$$\eta_s \equiv \eta[s_s(t), t] = \xi_i + \dot{\xi} t \tag{7}$$

where ξ_i denotes an initial base level elevation. Beyond the end of the alluvial zone the sediment forms a subaqueous foreset that progrades at slope S_a by avalanching. The bed profile on the foreset is

$$\eta = \eta_s - S_a (x - s_s) \tag{8}$$

over the zone $s_s < x < s_b$. The following continuity condition holds at the foreset-basement break;

$$\eta_b \equiv \eta[s_b(t), t] = \eta_s - S_a(s_b - s_s) = \eta_{base}[s_b(t)] \quad (9)$$

The above equation can be differentiated with respect to time to yield the result

$$\dot{s}_b = \frac{S_a - S_s}{S_a - S_{base}}\dot{s}_s + \frac{1}{S_a - S_{base}}\left.\frac{\partial \eta}{\partial t}\right|_{s_s} \quad , \quad S_s = -\left.\frac{\partial \eta}{\partial x}\right|_{s_s} \quad (10a,b)$$

where S_s denotes the alluvial bed slope at the shoreline.

If the subaqueous delta is prograding outward, an integration of (3) across the foreset subject to the condition that q_s vanishes at the foreset-basement intersection yields the shock condition

$$\dot{s}_s = \frac{1}{S_a - S_s}\left(\frac{q_{ss}}{s_b - s_s} - \left.\frac{\partial \eta}{\partial t}\right|_{s_s}\right) \quad (11a)$$

where

$$q_{ss} \equiv q_s[s_s(t), t] \quad (11b)$$

(Kostic and Parker, 2003a; Swenson et al., 2000). In simple terms, (11a) indicates that the shoreline progrades outward ($\dot{s}_s > 0$) in proportion to the residual sediment load q_{ss} at the shoreline. In the event that the subaqueous delta does not prograde outward, (11) is replaced by the condition of vanishing sediment at the shoreline;

$$q_s[s_s(t), t] = 0 \quad (12a)$$

Differentiating the above equation with respect to time, it is found that

$$\dot{s}_s = -\left(\left.\frac{\partial q}{\partial t}\right|_{s_s}\right) \Big/ \left(\left.\frac{\partial q}{\partial x}\right|_{s_s}\right) \quad (12b)$$

This condition corresponds to autoretreat of the shoreline if $\partial q/\partial x$ and $\partial q/\partial t$ are both negative at the shoreline.

It is now possible to specify regimes for delta development.

1. The case ($\dot{s}_s > 0$, $\dot{s}_b > 0$) corresponds to a normally prograding delta, i.e. one for which both the shoreline and the foreset-basement break undergo regression.
2. The case ($\dot{s}_s < 0$, $\dot{s}_b > 0$) corresponds to a delta with a retreating (transgressing) shoreline but prograding (regressing) foreset-basement break regresses. The case of autobreak is reached when s_b can no longer prograde due to sediment starvation.
3. The case ($\dot{s}_s < 0$, $\dot{s}_b = 0$) corresponds to autoretreat, for which the shoreline retreats (transgresses) and the subaqueous foreset becomes a dormant relict.

For cases 1 and 2 the appropriate downstream conditions at the shoreline are (7) and (11). For case 3 the appropriate conditions at the shoreline become (7) and (12).

The initial condition consists of a very short alluvial reach of specified constant slope, followed by a commensurately short foreset. The parameters s_u and s_s are assumed to take the initial values $s_u = 0$, $s_s = s_{si}$, so that the initial length of the fluvial zone is s_{si}. The initial elevation of the shoreline is taken to be $\eta_{si} = \xi_i = 0$, and the initial bed profile of the fluvial zone is

$$\eta(x, 0) = \eta_i(x) = S_{fi}(s_{si} - x) \; , \quad 0 < x < s_{si} \quad (13a)$$

where S_{fi} denotes the (constant) initial bed slope of the alluvial zone. The initial profile of the subaqueous delta is given as

$$\eta(x,0) = -S_a(x - s_{si}) , \quad s_{si} < x < s_{bi} \tag{13b}$$

The initial height of the foreset is taken to be $\Delta\eta_i$, so that

$$s_{bi} = s_{si} + \frac{\Delta\eta_i}{S_a} \tag{13c}$$

The basement profile $\eta_{base}(x)$ must thus have slope S_{base}, attain the elevation $S_{fi}s_{si}$ at x = 0 and attain the elevation $-\Delta\eta_i$ at $x = s_b$. This condition imposes a relation between $\Delta\eta_i$ and S_{base};

$$\Delta\eta_i = \frac{(S_{base} - S_{fi})s_{si}}{1 - \frac{S_{base}}{S_a}} \tag{14}$$

The basement profile, which is assumed to be invariant, is then given as

$$\eta_{base} = S_{fi}s_{si} - S_{base}x \tag{15}$$

The calculation proceeds as follows. The alluvial domain is discretized into N+1 nodes, where the last of these is located at the shoreline. For a given bed configuration S and q_s are computed on the alluvial zone from (1) and (2). The condition (4) is imposed in terms of a ghost node. The bed elevation one time step later is then computed from a discretized version of (3). The migration of the bedrock-alluvial break is computed from (6a). In the event that $q_{ss} > 0$, i.e. there is supply of sediment to the shoreline, the migration of the shoreline and foreset-basement break are computed from (11) and (10), respectively.

The aggradational nature of the problem forces q_s to decline in x from the bedrock-alluvial break s_u to the shoreline s_s. As the calculation proceeds, it can be expected that the shoreline value q_{ss} eventually drops to zero, indicating autobreak and the onset of autoretreat. In a discretized calculation, this is first realized in terms of a negative value of q_{ss}. From this point on conditions (10) and (11) must be abandoned and replaced with (12). The motion of the shoreline can be described by interpolating upstream from the current point s_s at which $q_{ss} < 0$ to a new value of s_s at which q_{ss} equals zero.

The above problem is solved on a deforming grid using the coordinate transformation

$$\bar{x} = \frac{x - s_u(t)}{s_s(t) - s_u(t)} , \quad \bar{t} = t \tag{16a,b}$$

Equations (3), (4), (6a), (7), (10a), (11a) and (12a) transform to the respective forms

$$\frac{\partial\eta}{\partial\bar{t}} - \frac{\left[\bar{x}\dot{s}_s + (1-\bar{x})\dot{s}_u\right]}{s_s - s_u}\frac{\partial\eta}{\partial\bar{x}} = -\frac{1}{s_s - s_u}\frac{\partial q_s}{\partial\bar{x}} \tag{17}$$

$$q_s\big|_{\bar{x}=0} = q_{sf} \tag{18}$$

$$\dot{s}_u = -\frac{1}{S_{base}}\frac{\partial\eta}{\partial\bar{t}}\bigg|_{\bar{x}=0} \tag{19}$$

$$\eta[1,\bar{t}] = \dot{\xi}\bar{t} \tag{20}$$

$$\dot{s}_b = \frac{1}{S_a - S_{base}}\left(S_a\dot{s}_s + \frac{\partial\eta}{\partial\bar{t}}\bigg|_{\bar{x}=1}\right) \tag{21}$$

$$\dot{s}_s = \frac{1}{S_a}\left(\frac{q_s(1,\bar{t})}{s_b - s_s} - \frac{\partial\eta}{\partial\bar{t}}\bigg|_{\bar{x}=1}\right) \tag{22}$$

$$q_s(1,\bar{t}) = 0 \tag{23}$$

NUMERICAL IMPLEMENTATION

The alluvial domain [0, 1] is discretized into N intervals each with length

$$\Delta\bar{x} = \frac{1}{N} \tag{24}$$

The upstream and downstream elevations are η_1 and η_{N+1}, respectively. The load nodes are staggered a distance of 0.5 $\Delta\bar{x}$ from the elevation nodes. The load node 1 corresponds to a ghost node where the sediment feed rate q_{sf} is specified. Equation (17) is discretized to

$$\eta_i = \eta_i - \left[\bar{x}_i\dot{s}_s + (1-\bar{x}_i)\dot{s}_u\right]S_i\Delta\bar{t} + \frac{q_{s,i} - q_{s,i+1}}{\Delta\bar{x}}\frac{\Delta\bar{t}}{s_s - s_u} \tag{25}$$

where $\Delta\bar{t}$ denotes the time step and

$$S_i = \begin{cases} \dfrac{1}{s_s - s_u}\dfrac{\eta_{i-1} - \eta_{i+1}}{2\Delta\bar{x}}, & i = 2..N \\ \dfrac{1}{s_s - s_u}\dfrac{\eta_i - \eta_{i+1}}{\Delta\bar{x}}, & i = 1 \end{cases} \tag{26}$$

The sediment transport rates $q_{s,i}$, i = 2..N+1 are computed as functions of $\hat{S}_i$ according to (1), where

$$\hat{S}_i = \frac{1}{s_s - s_u}\frac{\eta_{i-1} - \eta_i}{\Delta\bar{x}} \tag{27}$$

Conditions (19), (21) and (22) are easily implemented in discretized form to determine $\dot{s}_u$, $\dot{s}_s$ and $\dot{s}_b$. The terms $\left(\partial\eta/\partial\bar{t}\right)\big|_{\bar{x}=0}$ and $\left(\partial\eta/\partial\bar{t}\right)\big|_{\bar{x}=1}$ contained in these relations can be eliminated with the use of (25), so avoiding the need for iteration in a simulataneous solution of the evolution of bed elevation and boundary migration. For example, it is found between (19) and (25) that

$$\eta_1 = \eta_1 + \frac{S_{base}}{S_{base} - S_1}\frac{q_{sf} - q_{s,2}}{\Delta\bar{x}}\frac{\Delta\bar{t}}{s_s - s_u} \tag{28}$$

Condition (23) can be implemented via interpolation. It is seen from (1) that q_{ss} vanishes when S_s vanishes. Suppose that at some stage in the calculation $S_s \cong \hat{S}_{N+1}$ barely becomes negative. As long as the time step is sufficiently short $\hat{S}_N$ can be expected to be positive. As a result the position $s_{s,noload}$ corresponding to vanishing load at which (23) is satisfied can be estimated as

$$s_{s,noload} = s_u + (s_s - s_u)\left[\bar{x}_N - \frac{\Delta\bar{x}}{\hat{S}_{N+1} - \hat{S}_N}\hat{S}_N\right] \tag{29}$$

The value $s_{s,noload}$ is used as the value of s_s in the next time step, and η_i for the next time step is computed via (25) using the following estimate of $\dot{s}_s$;

$$\dot{s}_s = \frac{s_{s,noload} - s_s}{\Delta\bar{t}} \tag{30}$$

COMPUTATIONS AND COMPARISON AGAINST DATA

A sample calculation is given in Figure 5. The calculation is based on the second experimental run in Table 1 of Muto (2001), for which θ_{base} = 12.5°, $\dot{\xi}$ = 0.151 mm/s, q_{sf} = 0.904 cm^2/s and q_w = 4.36 cm^2/s. The shoreline is seen to first regress (prograde), then transgress (retrograde) and finally go into autoretreat transgression, i.e. the same pattern as illustrated in Figure 2.

The following parameters are chosen in order to compare the performance of the numerical model against all 29 runs of Table 1 of Muto (2001). The length of the alluvial reach at the end of the run L_{ae} is defined as

$$L_{ae} = s_s(t_{end}) - s_u(t_{end}) \tag{31}$$

where t_{end} denotes the end run time. Let s_{auto} and t_{auto} denote the distance to the point at which autobreak occurs and the time of autobreak, respectively. The elevation of the autobreak point above the bedrock basement η_{ab} is given as

$$\eta_{ab} = \eta(s_{auto}, t_{auto}) - \eta_{base}(s_{auto}) \tag{32}$$

In Figures (6), (7) and (8) computed values of L_{ae}, t_{auto} and η_{ab} are compared against measured values for all the runs of Table 1 of Muto (2001). The agreement is by no means perfect; the discrepancy is likely mostly due to the inadequacies of the rather simple sediment transport formulation of (1). In particular, the lack of a threshold condition for the transport of sediment in (1) likely causes an overprediction of both the time to autobreak t_{auto} and

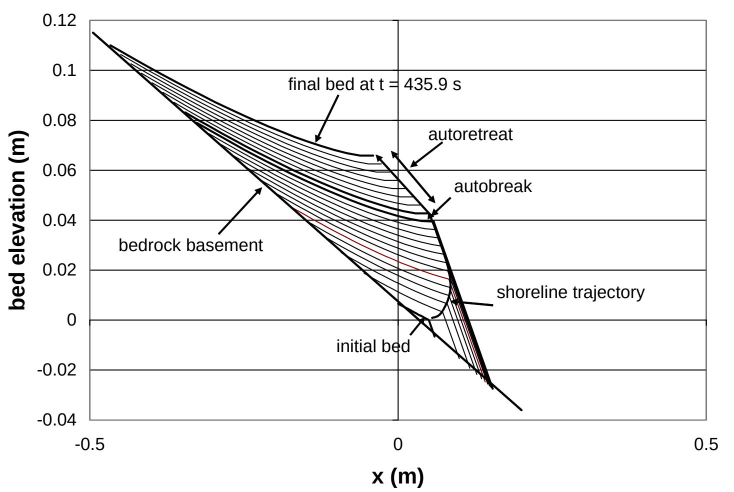

Figure 5. Simulation of the second run of Table 1 of Muto (2001) with the numerical model.

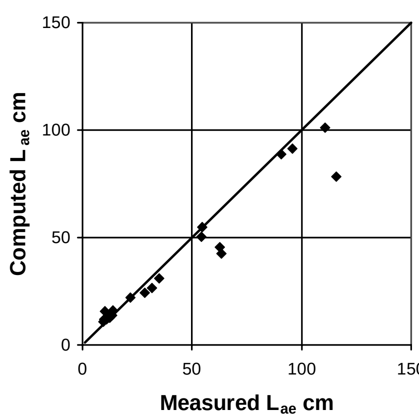

Figure 6. Computed versus measured values of L_{ae}.

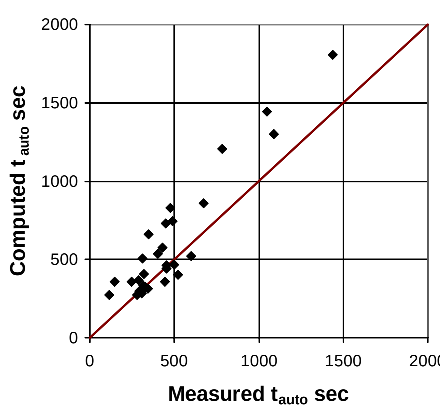

Figure 7. Computed versus measured values of t_{auto}.

the elevation at autobreak above bedrock η_{ab}. It is clear, however, that the numerical model captures the basic trends of the experiments without gross error.

FUTURE DEVELOPMENTS

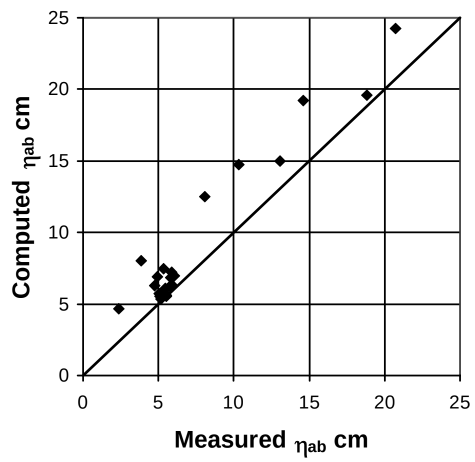

Figure 8. Computed versus measured values of η_{ab}.

The above formulation is designed specifically to capture autobreak and autoretreat as described by the experiments of Muto (2001). In these experiments the bed slope was high, the depth was on the order of a millimeter and the Froude numbers of the flow were either highly subcritical or supercritical. These conditions allow the neglect of backwater effects in describing delta dynamics. It is unlikely that such effects are negligible at field scale, where the Froude numbers are usually well into the subcritical range. In addition, a more realistic description of flow mechanics, including bed resistance, and sediment transport is necessary for field application. Kostic and Parker (2003a,b) have implemented these features in describing 1D deltas. In their analysis, however, the upstream end of the alluvial reach is not allowed to migrate upstream. Finally, an extension of the present 1D model to a 2D configuration including transverse flare of the fan is desirable.

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